The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 6, 2005

Course: EE 313 Evans

Name: Evans, Brian
Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise.**

Problem	Point Value	Your score	Topic
1	20		Differential Equation
2	20		Stability
3	24		Tapped Delay Line
4	24		Continuous-Time System Responses
5	12		Potpourri
Total	100		

K-30

Problem 1.1 Differential Equation. 20 points.

For a continuous-time system with input $x(t)$ and output $y(t)$ governed by the differential equation

$$\frac{d^2}{dt^2}y(t)+4\frac{d}{dt}y(t)+3y(t)=x(t)$$

for $t \geq 0^+$.

(a) What are the characteristic roots of the differential equation? 4 points.

The characteristic polynomial is $\lambda^2+4\lambda+3$.

$$\lambda^2+4\lambda+3=0$$

$$(\lambda+1)(\lambda+3)=0$$

The roots are $\lambda=-1$ and $\lambda=-3$.

(b) Find the zero-input response assuming non-zero initial conditions. Please leave your answer in terms of C_1 and C_2. 8 points.

$y_0(t) = C_1 e^{-t} + C_2 e^{-3t}$ for $t \geq 0^+$.

$y_0'(t) = -C_1 e^{-t} - 3C_2 e^{-3t}$ for $t \geq 0^+$.

(c) Find the zero-input response for the initial conditions $y(0^+) = -4$ and $y'(0^+) = 0$. 8 points.

$y(0^+) = C_1 + C_2 = -4$

$y'(0^+) = -C_1 - 3C_2 = 0 \Rightarrow C_1 = -3C_2$

Substituting $C_1 = -3C_2$, $-3C_2 + C_2 = -4$

$-2C_2 = -4$

$C_2 = 2$

$C_1 = -6$

$y_0(t) = -6e^{-t} + 2e^{-3t}$ for $t \geq 0^+$

Problem 1.2 Stability. 20 points.

In this problem, the input signal is denoted by $x(t)$ and the output signal is denoted by the output signal $y(t)$.

(a) Is the system defined by $\frac{d^2}{dt^2}y(t)+4\frac{d}{dt}y(t)+3y(t)=x(t)$ asymptotically stable, marginally stable, or unstable? Why? 8 points.

This differential equation appeared in problem 1.
The characteristic roots are -1 and -3.
Since they have negative real parts, the zero-input response is asymptotically stable.

(b) Let K be a real-value constant. For what values of K is the following system asymptotically stable? $\frac{d^2}{dt^2}y(t)-K\frac{d}{dt}y(t)-(K+1)y(t)=x(t)$. Why? 8 points.

The characteristic roots are

$$\lambda^2 - K\lambda - (K+1) = 0$$

$$\lambda = \frac{K \pm \sqrt{K^2 - 4(1)(-(K+1))}}{2}$$

$$= \frac{K \pm \sqrt{K^2+4K+4}}{2} = \frac{K \pm \sqrt{(K+2)^2}}{2} = \frac{K \pm (K+2)}{2} = \{-1, K+1\}$$

For asymptotic stability,
$\underbrace{-1<0}_{\text{ALWAYS}}$ and $K+1<0$

$$\boxed{K<-1}$$

Double check: $(\lambda+1)(\lambda-(K+1)) = \lambda^2 - K\lambda - (K+1)$.

(c) *Either prove the following statement to be true, or give a counterexample to show that the following statement is false*: The output of an LTI system in resonance is always unstable. 4 points

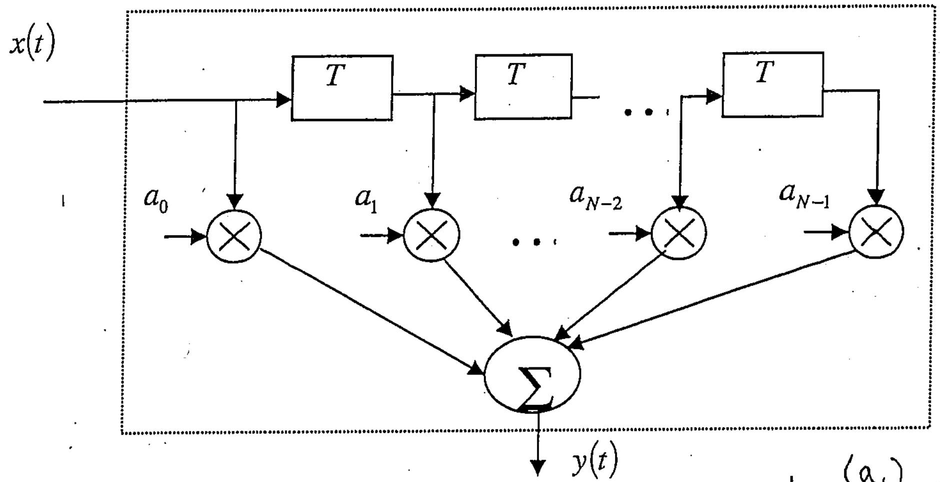
Consider the impulse response $h(t)=e^{\lambda t}u(t)$,
and the input signal $x(t)=e^{\lambda t}u(t)$ (slide 6-9).

$$y(t) = x(t) * h(t) = t\,e^{\lambda t}u(t)$$

This is an example of an output of an LTI system in resonance. If the real part of λ is negative, the output signal will decay to zero as $t \to \infty$. Hence, the statement is false.

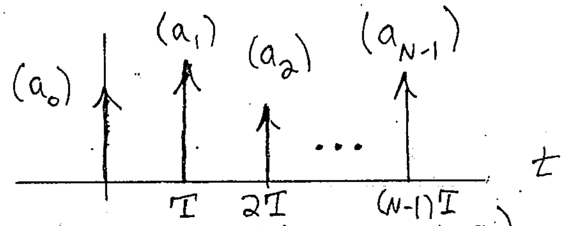
Problem 1.3 Tapped Delay Line. 24 points.

A linear time-invariant (LTI) continuous-time tapped delay line with input $x(t)$, output $y(t)$, and N-1 delay elements is shown below as a block diagram (from slide 2-4):



In the plot below, the notation (x) means that the area is X.

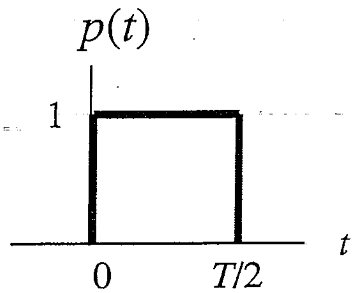
(a) Plot by hand the impulse response $h(t)$. 4 points.

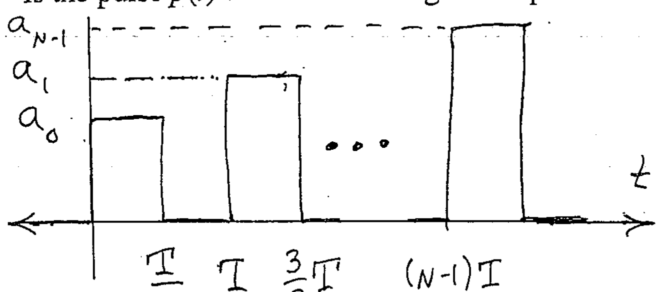


Impulse response is the response to an impulse. With $x(t) = \delta(t)$,

$h(t) = a_0 \delta(t) + a_1 \delta(t-T) + a_2 \delta(t-2T) + \cdots + a_{N-1} \delta(t-(N-1)T)$.

(b) Plot by hand the output $y(t)$ when the input $x(t)$ is the pulse $p(t)$ shown on the right. 10 points.

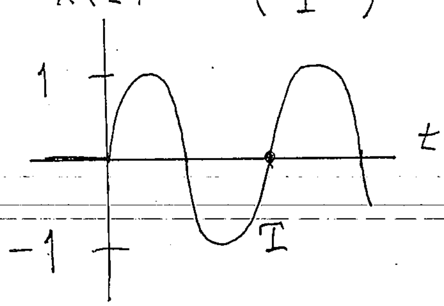


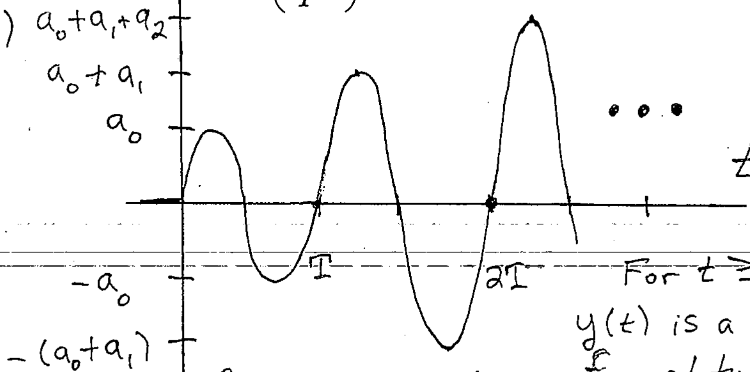


$y(t)$ has a finite duration $t \in [0, (N-\frac{1}{2})T]$.

(c) Plot by hand the output $y(t)$ when the input $x(t)$ is $\sin\left(\frac{2\pi}{T} t\right) u(t)$. 10 points.

$x(t) = \sin\left(\frac{2\pi}{T} t\right) u(t)$





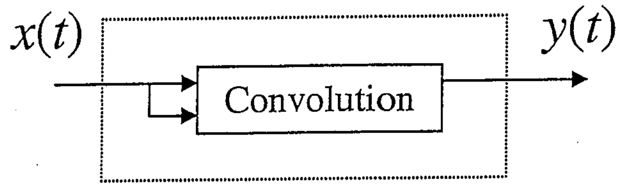
$y(t)$ is a one-sided signal of infinite extent.

For $t \geq (N-1)T$, $y(t)$ is a sine wave of amplitude $a_0 + a_1 + \cdots + a_{N-1}$.

K-33

Problem 1.4 Continuous-Time System Properties. 24 points.

Consider the continuous-time system with input $x(t)$ and output $y(t)$ that is shown on the right. The input-output relationship is



$$y(t) = x(t) * x(t)$$

where * means the convolution operation.

(a) Prove that the system has the linearity property, or give a counterexample that shows that the system does not have the linearity property. 12 points.

Input $x(t)$ gives output $y(t) = x(t) * x(t)$.

If we scale the input by a, then

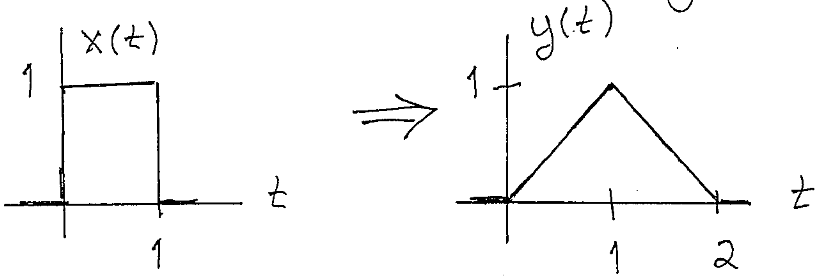
$$y_{scaled}(t) = (a x(t)) * (a x(t))$$
$$= a^2 \, (x(t) * x(t))$$
$$= a^2 \, y(t)$$

The system is not homogeneous, and therefore the system is not linear. An example is to show this is $x(t) = u(t)$. (The system is not additive.)

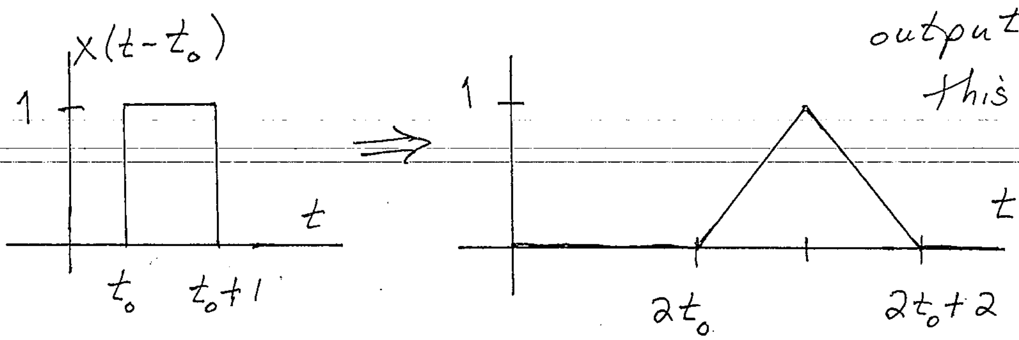
(b) Prove that the system has the time-invariant property, or give a counterexample that shows that the system does not have the time-invariant property. 12 points.

Input $x(t)$ gives output $y(t) = x(t) * x(t)$.

Let $x(t)$ be a causal rectangular pulse $\Pi(t - \frac{1}{2})$.



A shift of t_0 to the input yields a shift of $2t_0$ in the output. Therefore, this system is <u>not time-invariant</u>.



K-34

Problem 1.5 Potpourri. 12 points.

(a) *Either prove the following statement to be true, or give a counterexample to show that the following statement is false*: The continuous-time convolution of two finite duration signals always produces a finite duration result that is longer than either of the signals being convolved. 4 points.

Consider the following counterexample:

$$x(t) * \delta(t) = x(t)$$

If $x(t)$ is of finite duration, then the result of the convolution is not longer than itself.
Hence, the statement is false.
(Note: $\delta(t)$ can be considered a finite duration signal of zero length, but the value of the origin is undefined.)

(b) Give one signal processing or communication system that uses each of the following subsystems and describe the role that the subsystem plays in the function of the overall system:

i. Sinusoidal signal generator. 4 points.

- Amplitude modulation (slide 2-8). The sinusoidal generator is set to a frequency equal to the radio station frequency (f_c).
- Oscillator used to clock another subsystem, e.g. a analog-to-digital converter a digital-to-analog converter.

ii. Integrator. 4 points.

- Frequency modulation (slide 2-9). The integrator places a filtered version of the audio message signal ($m(t)$) into the time-varying phase of the FM signal.
- Analog-to-digital converter using sigma-delta modulation ~~(this hasn't been covered in EE313).~~

K-35